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EQUIVALENT TRANSFORMATIONS FOR MIXED LUMPED AND DISTRIBUTED CIRCUITS

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Abstract In this paper we show the basic concepts of the equivalent transformations for a class of mixed lumped and distributed circuits and extend these transformations to mixed lumped and multi-conductor coupled circuits. Then we discuss an equivalent transformation for a circuit consisting of a cascade connection of a lumped resonant circuit and unit element.

I. INTRODUCTION

It is well known that equivalent transformations are very powerful for analyzing and synthsizing the distributed transmission-line circuits and a large number of useful equivalent circuits have been recorded [1]-[11].

Many articles have discussed the analysis and synthesis of nonunifrom transmission lines and it has been shown that nonuniform transmission lines show superior responce than the ones of uniform transmission lines [12]-[15]. But it is quite difficult to find exact network functions of general nonuniform transmission lines from the telegrapher's equation except some nonunifrom transmission lines. Mixed lumped and distributed circuits also show superior responce and small-sized circuits can be obtained [16]-[19].

We have shown the equivalent transformations for a class of mixed lumped and distributed circuits by applying Kuroda's identities n-times and proceeding to the limit $n \to \infty$ [20]-[21]. By using these equivalent transformations, network functions of many nonuniform transmission lines can be obtained exactly and new design methods for mixed lumped and distributed circuits may be obtained.

In this paper, we first show the basic ideas of these equivalent transformations. Secondly we apply these ideas to m-port stub-circuits and multi-conductor coupled circuits and show the equivalent transformations for mixed lumped and coupled transmission line circuits. Finally we discuss the transformation for mixed lumped resonant circuit and unit element (UE).

II. EQUIVALENT TRANSFORMATIONS FOR MIXED LUMPED AND DISTRIBUTED CIRCUITS

Kuroda's identity can be applied n-times (n:integer) to the circuit shown in Fig.1(a), where line length and the characteristic admittance of the single open stub

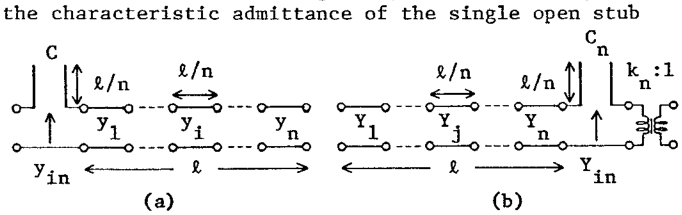


Fig.1. Kuroda's identity for the series open stub.

are ℓ/n and C, respectively, and line length and the characteristic admittances of the lossless uniform transmission lines (UE's) are ℓ/n and y_i (i=1,2,--,n). The transformed circuit is one consisting of a cascade connection of UE's with line length ℓ/n, a single open stub and an ideal transformer as shown in Fig.1(b). The element values of the transformed circuit are given as follows:

$$k_j = 1 + \frac{1}{C}\sum_{i=1}^{j} y_i \qquad (j=1,2,--,n) \tag{1}$$

$$k_0 = 1 \tag{2}$$

$$Y_j = \frac{y_j}{k_{j-1}\,k_j} \qquad (j=1,2,--,n) \tag{3}$$

$$C_n = \frac{C}{k_n}. \tag{4}$$

For the simplicity, by setting

$$y_i = y_j \equiv y \qquad \text{(for all i,j)} \tag{5}$$

we get

$$Y_j = \frac{y}{(1+\frac{i-1}{n}\frac{y}{C_0})(1+\frac{i}{n}\frac{y}{C_0})} \qquad (j=1,2,--,n) \tag{6}$$

$$C_n = \frac{nC_0}{1+\frac{y}{C_0}} \tag{7}$$

$$k_n = 1 + \frac{y}{C_0} \equiv k \tag{8}$$

where

$$C = nC_0 . \tag{9}$$

The characteristic admittances Y_i of the UE's are discrete values decreasing monotonically. The coordinates x of the i-th UE is given by

$$x = \frac{i}{n}\ell. \tag{10}$$

By substituting (10) in (6) and proceeding to the limit $n \to \infty$ in such a way that $C \to \infty$ but C_0 remains finite, then we obtain

$$\lim_{n\to\infty} Y_j = \lim_{n\to\infty} \frac{y_j}{k_{j-1}\,k_j} = \frac{y}{(1+\frac{y}{C_0}\frac{x}{\ell})^2} \equiv \frac{y}{k(x)^2}$$
$$= Y(x). \tag{11}$$

Y(x) is the characteristic admittance distribution of the parabolic tapered transmission line. Namely, if Kuroda's identity are applied a number of times, the characteristic admittance distribution of the transformed circuit turns from a discrete function of the distance x into a continuous function. Under this condition, the admittance y_{in} and Y_{in} of the single open stubs shown in Fig.1(a) and (b), respectively, yield

$$\lim_{n\to\infty} y_{in} = \lim_{n\to\infty}\left[jnC_0 \tan\frac{\beta\ell}{n}\right] = jC_0\beta\ell \tag{12}$$

$$\lim_{n\to\infty} Y_{in} = \lim_{n\to\infty}\left[j\frac{nC_0}{k} \tan\frac{\beta\ell}{n}\right] = j\frac{C_0}{k}\beta\ell \tag{13}$$

where β is the phase constant. Namely, the single open stubs become lumped capacitors.

Here, we define the characteristic admittance of the i-th UE of the circuit shown in Fig.1(a) as follows:

$$y_i = Y_0\left[1 + \frac{a_1}{n}(i-1) + \frac{a_2}{n^2}(i-1)^2 + \cdots\right. \tag{14}$$

where Y_0 is the characteristic admittance of the first UE and a_m (m=1,2,--) are constants. By setting coefficients a_m appropriate values and proceeding to the limit $n \to \infty$, we can obtain the various characteristic admittance distribution Y(x) of nonuniform transmission line from (14)

$$Y(x) = \lim_{n\to\infty} y_i = Y_0\left[1 + a_1\left(\frac{x}{\ell}\right) + a_2\left(\frac{x}{\ell}\right)^2 + \cdots + a_m\left(\frac{x}{\ell}\right)^m + \cdots\right]. \tag{15}$$

By using the same techniques described above, we obtain

$$k(x) = \lim_{n\to\infty} k_j = 1 + \frac{Y_0}{C_0}\left[\left(\frac{x}{\ell}\right) + \frac{a_1}{2}\left(\frac{x}{\ell}\right)^2 + \cdots + \frac{a_m}{m+1}\left(\frac{x}{\ell}\right)^{m+1} + \cdots\right] = 1 + \frac{1}{C_0}\int_0^{\frac{x}{\ell}} y(\lambda)\, d\left(\frac{\lambda}{\ell}\right) \tag{16}$$

$$k = k(x)\big|_{x=\ell} \tag{17}$$

and

$$Y(x) = \lim_{n\to\infty} Y_j = \lim_{n\to\infty} \frac{y_j}{k_{j-1}\,k_j} = \frac{y(x)}{k(x)^2}. \tag{18}$$

Namely, we obtain equivalent transformation for mixed lumped and nonuniform distributed circuit shown in Fig.2.

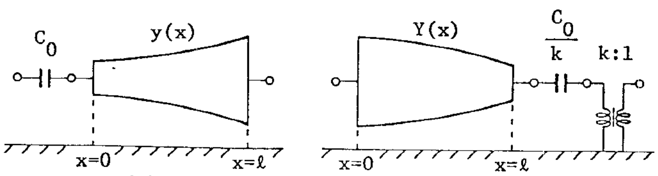
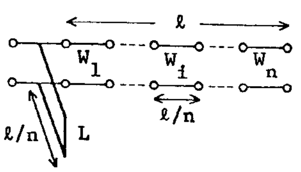
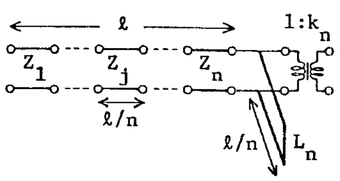


Fig.2. Equivalent transformation for mixed lumped and nonuniform distributed circuit.

In the same manner, we can apply Kuroda's identity n-times to the circuit shown in Table I, where L and W_i (i=1,2,--,n) are the characteristic impedances of the single short stub and UE's, respectively. By setting $L = nL_0$ and proceeding to the limit $n \to \infty$, we obtain the equivalent transformation shown in Table II.

Original circuit	Equivalent circuit
	

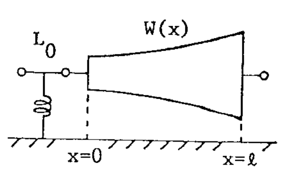
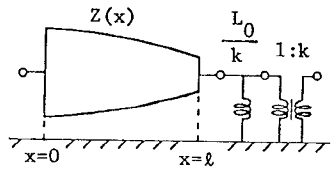
Formula

$$k_j = 1 + \frac{1}{L}\sum_{i=1}^{j} W_i \quad (j=1,2,--,n) \quad , \quad k_0 = 1$$

$$Z_j = \frac{W_j}{k_{j-1}\,k_j} \quad (j=1,2,--,n) \quad , \quad L_n = \frac{L}{k_n}$$

Table 1.

From circuits in Fig.2 and Table II, the equivalent circuits of the transformed nonuniform transmission lines with Y(x) and Z(x) can be expressed as the mixed lumped and nonuniform distributed circuits. Therefore, if an exact network function of an original nonuniform transmission line is given, a network function of a transformed nonuniform transmission line can be derived exactly.

Original circuit	Equivalent circuit
	

Formula

$$k(x) = 1 + \frac{1}{L_0}\int_0^{\frac{x}{\ell}} W(\lambda)\, d\left(\frac{\lambda}{\ell}\right) \ , \ k = k(x)\big|_{x=\ell} \ , \ Z(x) = \frac{W(x)}{k(x)^2}$$

Table II.

III. EQUIVALENT TRANSFORMATIONS FOR MIXED LUMPED AND MULTI-CONDUCTOR COUPLED CIRCUITS

The techniques shown in the preceding section can be applied to m-port networks consisting of stub circuits and multi-conductor coupled transmission lines (CTL's), under certain condition.

We define the m-port circuit consisting of short-circuited stubs (SCS) in Fig.3(a), and the m-port circuit consisting of open-circuited stubs (OCS) in Fig.3 (b). Here, $L_{ij}^{(I)}$ and $C_{ij}^{(I)}$ (i,j=1,2,--,m ; I=0,1,--,n) are characteristic impedances of short stubs of length ℓ/n and characteristic admittances of open stubs of length ℓ/n, respectively.

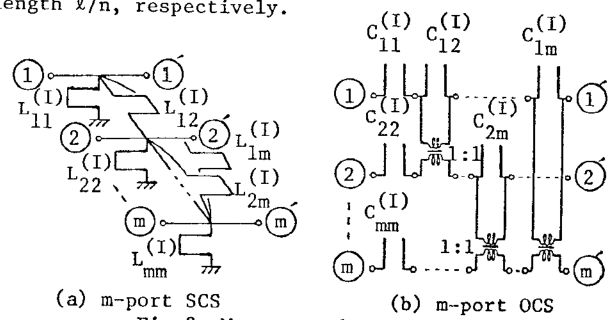


Fig.3. M-port stub circuits.

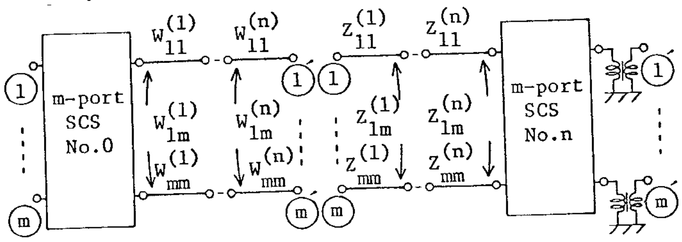
Distributed circuits consisting of the m-port SCS and multi-conductor cascaded transmission lines, shown in Fig.4(a), are equivalent to ones consisting of cascade connections of multi-conductor cascaded transmission lines whose characteristics impedances are differnt from original ones, an m-port SCS, and an m-port ideal transformer bank, shown in Fig.4(b), under the following conditions:

$$\left[\frac{1}{L^{(I-1)}}\right]\left[W^{(I)}\right] = A\,[E] \quad (I=1,2,--,n). \tag{19}$$

Where, $\left[\frac{1}{L^{(I)}}\right]$ is the m×m matrix defined as

$$\left[\frac{1}{L^{(I)}}\right] = \begin{bmatrix} \left(\sum_{i=1}^{m}\frac{1}{L_{1i}^{(I)}}\right) & \cdots & -\frac{1}{L_{1m}^{(I)}} \\ \vdots & & \vdots \\ -\frac{1}{L_{m1}^{(I)}} & \cdots & \left(\sum_{i=1}^{m}\frac{1}{L_{mi}^{(I)}}\right) \end{bmatrix}. \tag{20}$$

and [$W^{(I)}$] is the m×m characteristic impedance matrix of I-th unit section of the m-wire CTL, [E] is the m×m identity matrix, and A is the constant.



(a) original circuit. (b) equivalent circuit.
Fig.4. Equivalent transformation

Element values of the eqivalent circuit shown in Fig.4 (b) are obtained as follows:

$$k^{(I)} \equiv k_1^{(I)} = k_2^{(I)} = --- = k_m^{(I)}$$

$$= 1 + \sum_{j=1}^{m} \left(\frac{1}{L_{jj}^{(0)}} \sum_{r=1}^{I} W_{ij}^{(r)} \right) \qquad (21)$$

$$Z_{ij}^{(I)} = \frac{W_{ij}^{(I)}}{k^{(I-1)}\, k^{(I)}} \qquad (22)$$

$$L_{ij}^{(I)} = \frac{L_{ij}^{(0)}}{k^{(I)}} \qquad (i,j=1,2,--,m \ ; \ I=1,2,--,n). \qquad (23)$$

In the same manner as Section II, we set

$$x = \frac{I}{n}\ell \qquad (24) \qquad\qquad L_{ij}^{(0)} = nL_{ij}. \qquad (25)$$

By using (24) and (25) and proceeding to the limit $n\to\infty$, we can obtain the equivalent transformation for mixed lumped and multi-conductor coupled circuits shown in Fig.5. The element values of the transformed circuit are given as follows:

$$Z_{ij}(x) = \lim_{n\to\infty} Z_{ij}^{(I)} = \frac{W_{ij}(x)}{k(x)^2} \qquad (i,j=1,2,--,m) \qquad (26)$$

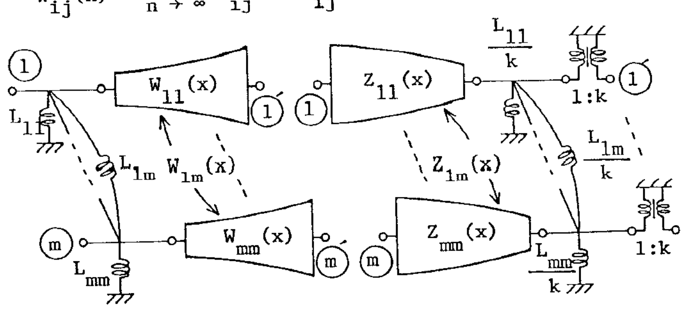
$$k(x) = \lim_{n\to\infty} k^{(I)} = 1 + \sum_{j=1}^{m} \left[\frac{W_{ij}^{(0)}}{L_{jj}} \int_0^{\frac{x}{\ell}} f(\lambda)\, d\!\left(\frac{\lambda}{\ell}\right) \right] \qquad (27)$$

(i=1,2,--,m)

$$k = k(x)\big|_{x=\ell}. \qquad (28)$$

Where, f(x) is a nonuniform taper function defined as

$$W_{ij}(x) = \lim_{n\to\infty} W_{ij}^{(I)} = W_{ij}^{(0)} f(x). \qquad (29)$$



(a) original circuit. (b) equivalent circuit.
Fig.5. Equivalent transformation

Similarly, the equivalent transformation for the circuit consisting of a cascade connection of the m-port OSC and multi-conductor nonuniform coupled circuit, the dual of the circuit shown in Fig.5, may be obtained.

IV. EQUIVALENT REPRESENTATION OF MIXED LUMPED RESONANT AND DISTRIBUTED CIRCUITS

It is interesting to make equivalent representations of the circuits consisting of the cascade connection of general lumped circuits and UE clear. Here, we suggest that the equivalent circuit of the circuit consisting of of the cascade connection of a lumped resonant circuit and UE may be expressed with the circuit consisting of cascade connection of nonuniform transmission line and the lumped Brune section as shown in Fig.6.

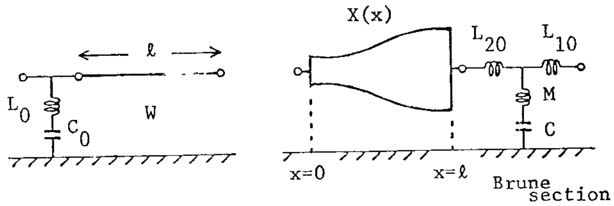


Fig.6 Transformation for mixed lumed resonant circuit and UE.

This equivalency may be derived by applying a transformation for the distributed Brune section and UE again and again and using a similar procedure described in section II. But it is very difficult to find the characteristic impedance distribution of the transformed nonuniform transmission line analytically. Some numerical examples of X(x) are shown in Fig.7.

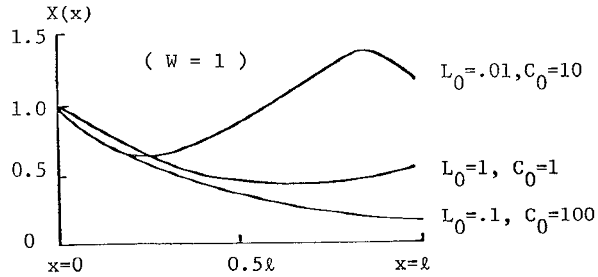


Fig.7 Examples of X(x)

V. CONCLUSION

We showed the concept of the equivalent transformations for mixed lumped and distributed circuits based on Kuroda's identity and applied these to mixed lumped and muliti-conductor coupled circuits.

REFERENCES

1.R.Sato: Joint Conv. Rec. of Elec. and Electron Engrs. of Japan, 1947
2.———: Joint Conv. Rec. of Comm. Engrs. of Japan, 1947
3.K.Kuroda: text for lectures at Joint Meeting of Kansai Branch of IECE, Japan, p32, 1952
4.H.Ozaki and J.Ishii: IRE Trans.CT, CT-2, p325, 1955
5.N.Ikeno: IRE Trans.CT, CT-4,p22, 1957
6.H.Ozaki and J.Ishii: IRETrans.CT, CT-5, p105, 1958
7.E.M.J.Jones and J.T.Bolljahn: IEEE Trans.MTT, MTT-4, p75, 1956
8.E.G.Cristal: IEEE Trans.MTT, MTT-14, p296, 1966
9.K.K.Pang: IEEE Trans.CT, CT-13, p235, 1966
10.R.Sato and E.G.Cristal: IEEE Trans.MTT, MTT-18, p122, 1970
11.R.Sato: IEEE Trans.MTT, MTT-19, p431, 1971
12.H.Kaufman: IRE Trans.AP-3, p218, 1955
13.H.Berger: IEEE Trans.CT, CT-13, p92, 1966
14.R.W.Klopfenstein: Proc. IRE, vol.44, p31, 1956
15.G.Sugawara, Y.Nemoto and R.Sato: TransIECE Japan, J60-A p956, 1977
16.R.Levy: IEEE Trans.MTT, MTT-18, p1064, 1970
17.———: IEEE Trans.MTT, MTT-20, p223, 1972
18.O.P.Gupta: IEEE Trans.CT, CT-20, p161, 1973
19.K.Kobayashi, Y.Nemoto and R.Sato: Trans., IECE Japan, J64-A p23, 1981
20.———: IEEE Trans.MTT, MTT-29, p81, 1981
21.———: IEEE Trans.MTT, MTT-30, p140, 1982